

A study of McVittie Spacetime in Stationary and Dynamical Regime

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One of the solutions of the Einstein equations, called McVittie solution, signifying a black hole embedded by the dynamic spacetime is studied. In the stationary spacetime the McVittie metric becomes the Schwarzschild-de Sitter metric (SdS). The geodesic of a freely falling test particle towards the black hole is examined in the SdS spacetime. It is found that unlike the Schwarzschild case the potential of such particle becomes maximum at a point where it eventually stops for a while and then resumes its motion towards the center of the black hole. It is shown that an observer or system of particles is spaghettified near the black hole singularity in the SdS spacetime. The dynamics of the universe in the framework of McVittie metric, being a generalized time dependent SdS solution, is represented in terms of that point, called stationary or turning point. The motion of the stationary point is studied in various regimes of the expanding universe and the possible outcomes are discussed in brief.

1. Introduction

Finding solutions to Einstein's field equations that describe a physically viable system have always been challenging. One of the main reasons for it is the highly non-linear nature of these equations. Describing the dynamics of objects like compact stars require interior solutions of Einstein's field equations, as well as for finding black hole solutions, physically acceptable solutions for black holes are essential. If we look into the history of Einstein's general theory of relativity (GR), there are two exact spherically symmetric chargeless non-rotating solutions obtained that have been studied in various contexts for several years.

One is the Schwarzschild solution that can describe the nature of the gravitational field residing outside a chargeless spherical mass with rotational velocity and cosmological constant taken to be zero. For several years this solution was vastly used to theoretically probe the nature of spacetime outside compact stars and black holes. But, the problem was that this solution doesn't consider the expanding nature of the universe in the background with the existence of mass. However, the other exact solution named Friedmann-Robertson-Walker-Lemaître (FRWL) metric^[1,2] assumes the spatial component of the metric to be time-dependent and the space to be isotropic and homogeneous. It is also noticeable from the literature survey that Faraoni^[3] and Jacques^[4] took the effects of the expanding universe and the cosmological impact of this expansion on the local system. For gravitationally bound systems, they also studied the nature of local attraction and analyzed black holes embedded in the spherical cosmological background in the framework of general relativity. Using a time-dependent spacetime, Arakida^[5] analyzed the dominant effects that come from the cosmological expansion.

At the beginning, let us emphasize the point here that we have opted for the present study on McVitty solution because of its interesting features in the static and dynamical regimes. As such the problem of dynamical black holes is not yet solved, so this study goes in that virgin line. In fact, it is known that McVitty black hole is almost unique and consistent with dynamical black holes. In this connection the work by Nojiri et al.^[6] is specifically worth mentioning where they have searched for dynamical black holes in various theories of gravity. They construct models of Einstein and $f(R)$ gravity with two scalar fields, which ad-

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